~~Basic~~ Structures for IIR Systems

The realization of IIR systems involve recursive computational algorithms

① Direct form Realization

The system function $H(z) = \dfrac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$

characterizes an IIR system & can be viewed as 2 systems in cascade i.e

$$H(z) = H_1(z) \cdot H_2(z)$$

where, $H_1(z)$ consists of zeros of $H(z)$
& $H_2(z)$ " " poles of $H(z)$

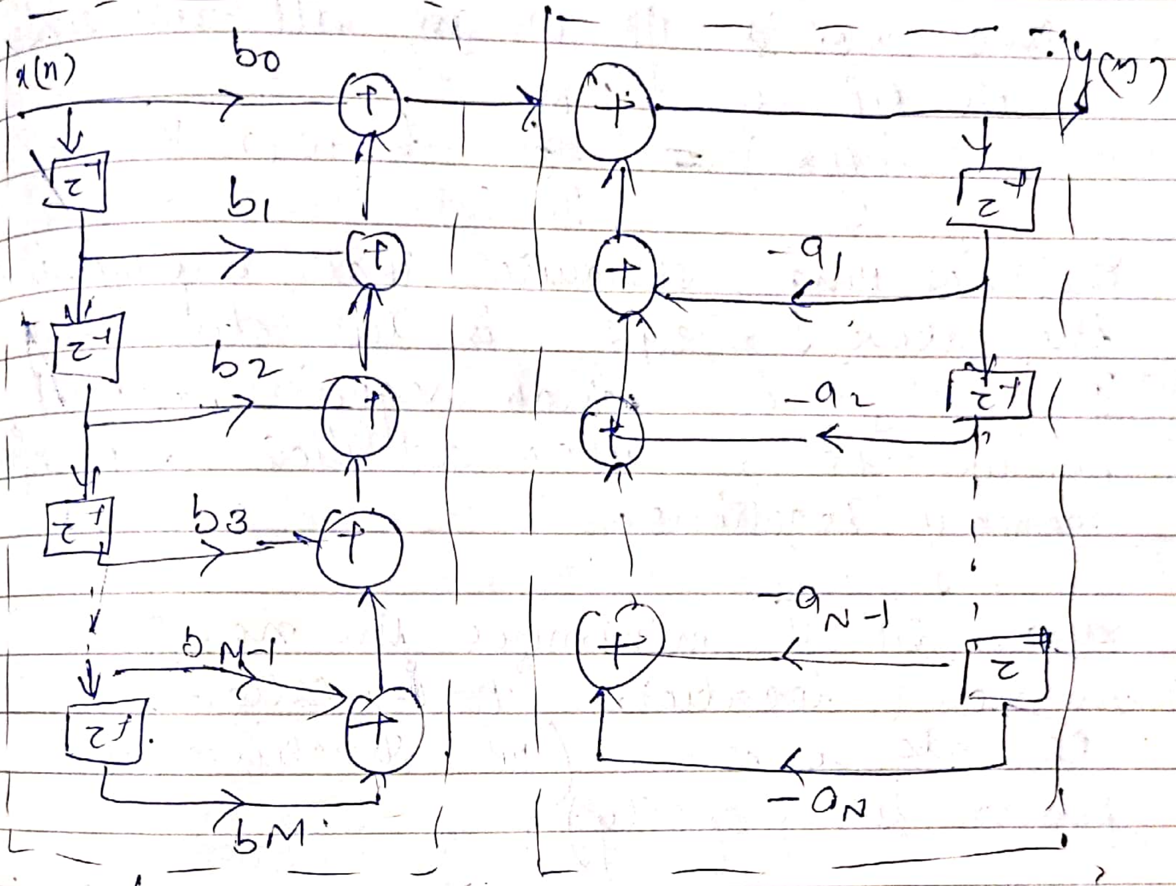
$$\therefore H_1(z) = \sum_{k=0}^{M} b_k z^{-k}$$

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

Since $H_1(z)$ is an FIR system we attach all-pole system ($H_2(z)$) in cascade with $H_1(z)$ to obtain Direct form I realization shown here

Diff eq (Recursive)

$$y(n) = -\sum_{k=0}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$



all zero system — All pole system

Direct form I Realization.

Requires M+N+1 multiplications, M+N additions & M+N+1 memory locations.

Now → if all pole-filter $H_2(z)$ is placed before all-zero filter, we gets Direct form-II realization.

We know

diff. eq for all-pole filter is

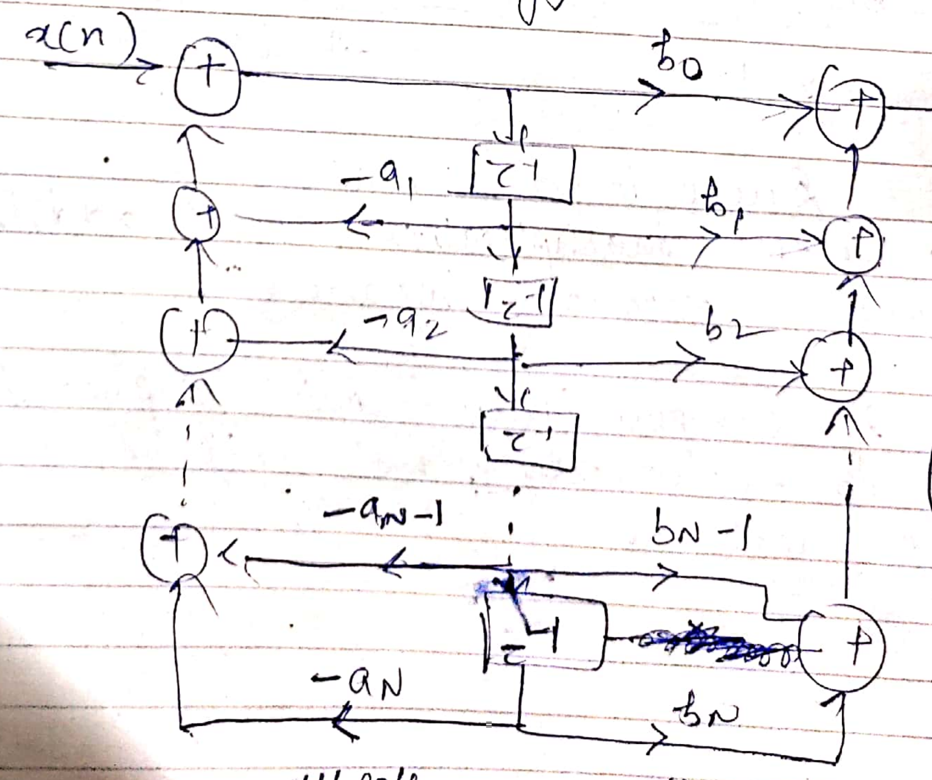
$$w(n) = -\sum_{k=1}^{N} a_k w(n-k) + x(n).$$

Since $w(n)$ is i/p to an all-zero system its o/p is

$$y(n) = \sum_{k=0}^{M} b_k w(n-k)$$

The resulting structure that implements the above 2 eqs is the direct form II realization which requires $M+N+1$ multiplication, $M+N$ addition & $\max\{M,N\}$ memory locations.

Since DF II minimizes the no. of memory locations it is called <u>canonic form</u>. (IIR structures also possess this property)



DF II realization (N=M)

all pole   all zero

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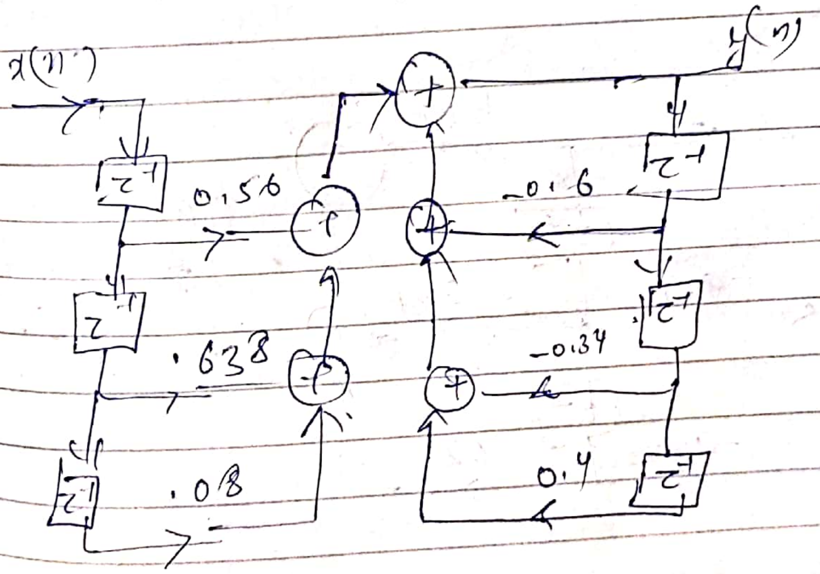
Q Det. DF I & II realization for 3rd order IIR Transfer function

$$H(z) = \frac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2}$$

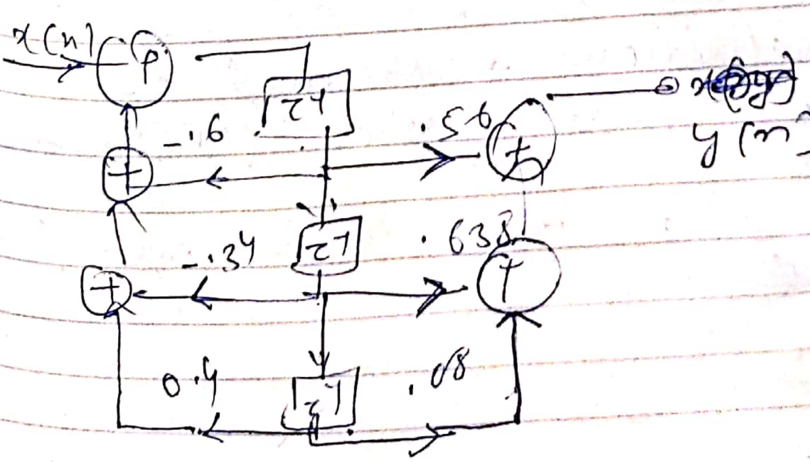
soln ÷ N & D by $2z^{-3}$ we get standard form of the transfer function as

$$H(z) = \frac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} + 0.4z^{-3}}$$

DF I → all zero & then all-pole
DF II → all pole & then all-zero.



DF I.



DF II

Scanned with CamScanner

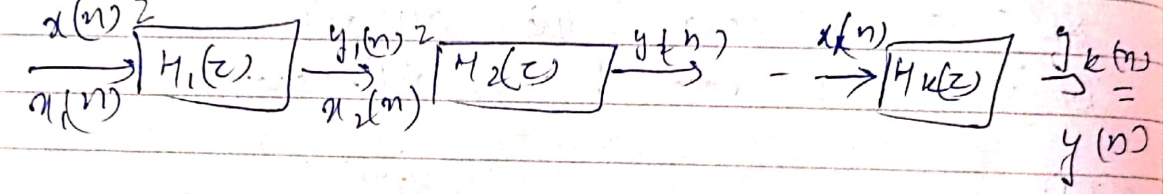
IIR Cascade form

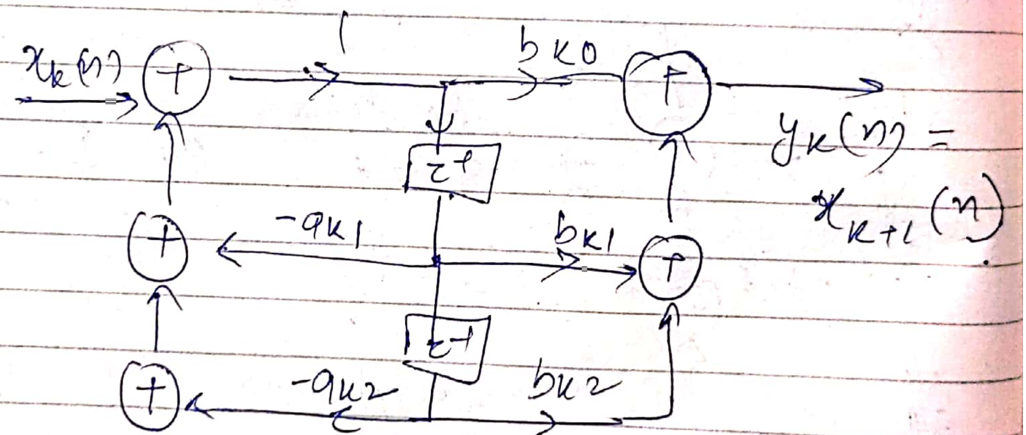
$$H(z) = \prod_{k=1}^{K} H_k(z)$$

$$H_k(z) = \frac{b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}}{1 + a_{k1} z^{-1} + a_{k2} z^{-2}}$$

the parameter b_0 can be distributed equally among k filter sections so that $b_0 = b_{10} b_{20} \cdots b_{k0}$

The coeff $\{a_{ki}\}$ & $\{b_{ki}\}$ r real



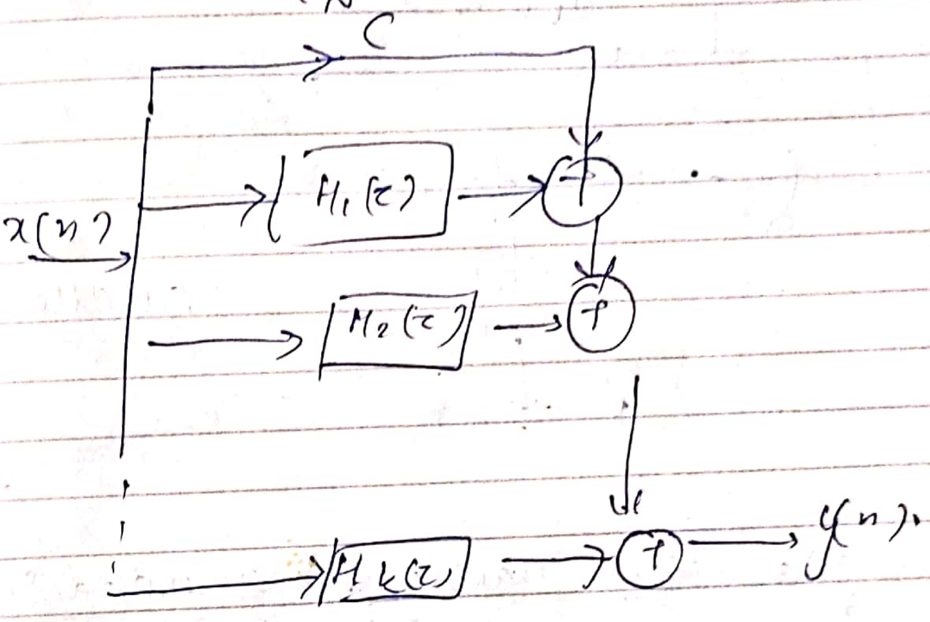


Parallel Realization → obtained by performing partial fraction expansion of H(z).

By performing partial fraction expansion of H(z)

$$H(z) = C + \sum_{k=1}^{N} \frac{A_k}{1-p_k z^{-1}}$$

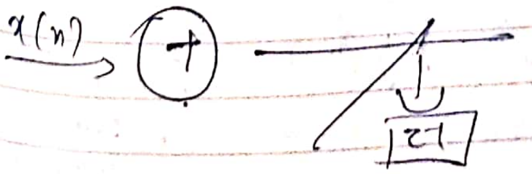
$\{p_k\}$ → poles, $\{A_k\}$ → coeff (residue)
& $C = \frac{b_N}{a_N}$

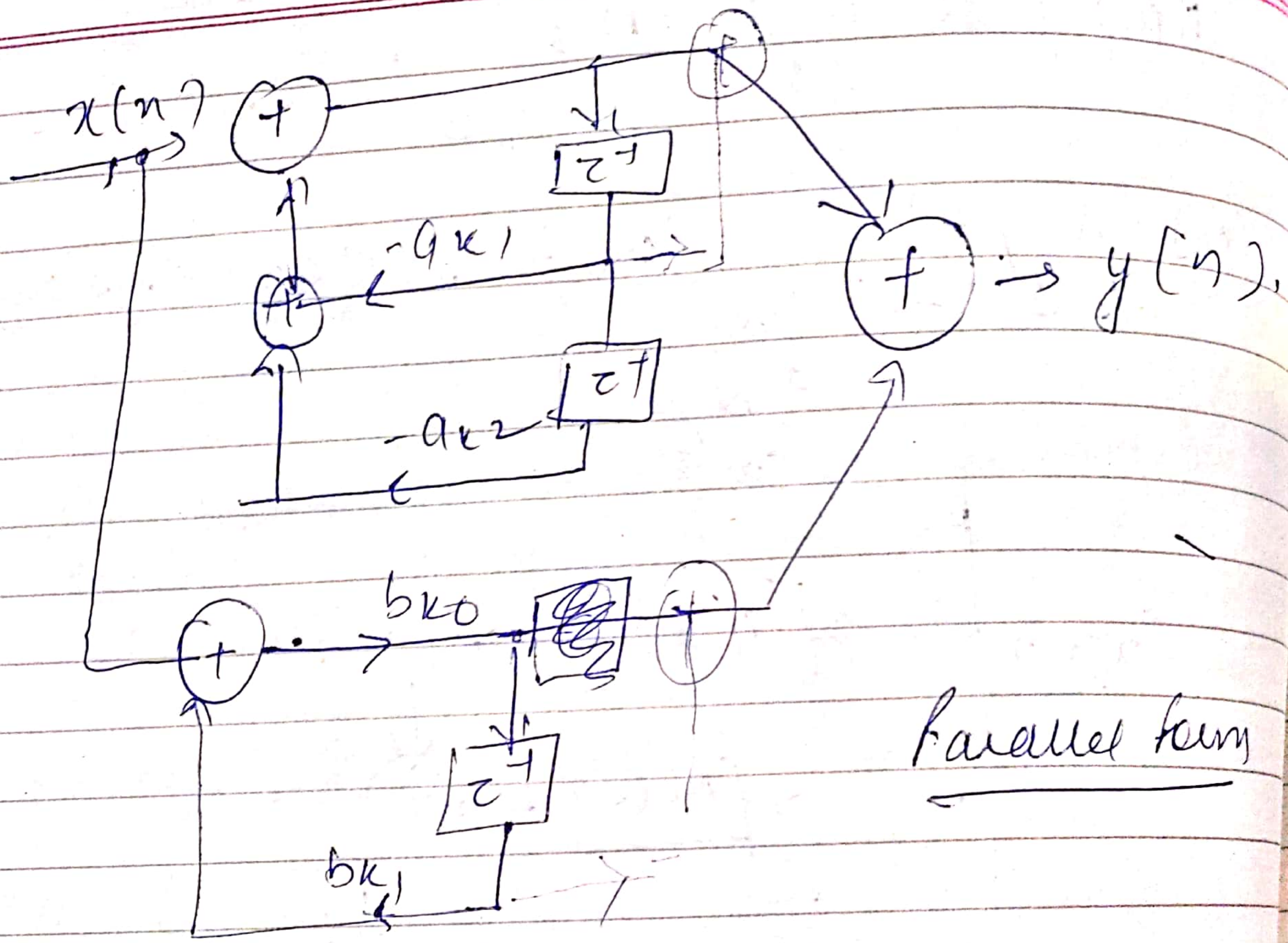


if $H_k(z) = \dfrac{b_{k0}+b_{k1}z^{-1}}{1+a_{k1}z^{-1}+a_{k2}z^{-2}}$

then $H(z) = C + \sum_{k=1}^{K} H_k(z)$

where K is integer part of $(N+1)/2$.
when N is odd, one of the $H_k(z)$ is really single-pole system i.e. $b_{k1} = a_{k2} = 0$





Parallel form